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Attitude Control of a Bias Momentum Geostationary Satellite in an Inclined Orbit

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An attitude control logic for a pitch bias momentum geostationary satellite with a skewed roll/yaw magnetic torquer in an inclined orbit is presented. The inclined orbit operation for geostationary satellites is primarily targeted to saving onboard fuel. A specific spacecraft model is used to establish operational strategies for the attitude maneuver in an inclined orbit. An onboard momentum wheel attached to a pivot steering mechanism for pitch and roll control and a skewed magnetic torquer for roll/yaw control are employed as main actuators. The control commands are issued at regular time intervals to reflect ground-loop implementations of the proposed technique. A real-time ground-based attitude estimator is designed to update the yaw angle information using roll angle telemetry data. Simultaneous pitch and roll/yaw pointing with the inclination angle of 2 deg turns out to be feasible by the proposed method.

I. Introduction

COMMUNICATION satellites at geostationary orbit are required to maintain zero or very small inclination angle about the equatorial plane.[1–6] This is due to the mission requirement of geostationary satellites, which stay at a stationary orbital location providing communication and broadcasting services. The small inclination angle is maintained by frequent execution of north/south stationkeeping maneuvers.[7,8] A significant amount of onboard fuel is consumed for the north/south stationkeeping maneuvers; thus, spacecraft mission life is usually limited by the amount of fuel available. Once the fuel is depleted, the inclination angle gradually increases, mainly caused by the sun/moon gravitational attractions. The nonzero inclination angle results in depointing of the antenna beam from the Earth target point. Thus, inclination angle is one of the key parameters in the mission operation of geostationary satellites. As long as the communication service is not affected, the acceptable tolerance of nonzero inclination angle extends the lifetime of a satellite through fuel saving.

The nonzero inclination angle causes north/south and east/west variations of the trace of a satellite seen from Earth.[7,8] To point a fixed location on Earth, the satellite in an inclined orbit needs attitude maneuvers over a 1-day orbit period. There are few examples of geostationary satellites operated in an inclined orbit accompanied by a certain level of inclination angle.[1,2] The COMSAT maneuver applied to the ARABSAT is a representative example of the inclined orbit operation.[2] The GSTAR III is also being operated in an inclined orbit,[1] and the KOREASAT F1 located at 116° east and launched in 1995 has recently started inclined orbit operation as well. The excessive consumption of onboard fuel for KOREASAT F1 for the final orbit raising resulted in the reduction of the spacecraft lifetime almost by 40% from the original design.

The central idea of the COMSAT maneuver is to utilize momentum wheel assemblies to tilt spacecraft body axes. The angular momentum vectors of different wheels are manipulated to create desired spacecraft offset attitude angles.[2] The roll and pitch angles are varied continuously along the orbital location of the satellite. The COMSAT maneuver was implemented using personal computer-based software that is interfaced with the ground system.[2] Parvez and Misra developed attitude maneuver schemes for GSTAR III operation.[1] In their approach, a momentum wheel with a pivot steering mechanism was used to control the roll angle variation that causes antenna boresight angle change. An onboard processor was combined with pivot steering commands to generate the roll and pitch offset pointing commands. Onboard thrusters were also employed to limit the maximum yaw angle at the nodal crossing point to minimize rf polarization plane change.[1]

In Refs. 3 and 4, taking rf signal measurements from the ground is considered to sense the spacecraft attitude to develop antenna pointing strategy. In particular, in Ref. 3 using radio signals for direct measurement of attitude is discussed, and the control algorithm is based on such measurement techniques. In Ref. 4, various sensors including sun sensors, Earth sensors, rate gyro, C-band interferometer, and communication monopulse subsystem are employed. The yaw angle is measured by an inertial reference unit and a Polaris sensor. The monopulse system and interferometer are adopted to measure attitude angles from ground stations.

The approach in Ref. 5 assumes a V-wheel-type bias momentum wheel whereas onboard closed-loop control is applied. Reference 6, on the other hand, employs an open-loop-type methodology for which the attitude offset angles are computed and transmitted to the spacecraft to maintain desired payload pointing directions.

In this study we present a new attitude maneuver technique for a pitch bias momentum geostationary satellite in an inclined orbit. The control law design goal is to develop an antenna pointing algorithm using onboard actuators. The spacecraft model is identical to GSTAR III, available in Ref. 1, and to KOREASAT F1, both of which are built using similar bus structures. However, the control strategies proposed in this study are different from those of Ref. 1. In fact the control logic is applicable to generic pitch bias momentum spacecraft with a pitch momentum wheel attached to a pivot mechanism and a skewed magnetic torquer for roll/yaw control.

The principal idea of the proposed control strategy is to take the attitude deviations due to orbit inclination angle as reference trajectories and to develop actuator commands that compensate for the attitude perturbation effects. The inclination angle used for this analysis is 2 deg. Given this inclination angle, the developed logic is effective, but higher values of inclination angle may be accommodated by the results of this study through simple hardware reconfiguration. The advantage of the proposed method is to use a magnetic torquer and a momentum wheel, which does not require onboard fuel consumption, for independent three-axis attitude control. The pitch

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and roll angles are controlled by the single momentum wheel with the pivot device, while the yaw angle is controlled by the magnetic torquer. This is a significant difference from the approach in Ref. 1, where the maximum yaw angle at the nodal crossing point is adjusted by thruster firings. The magnetic torquer as used in this study has wide applications in spacecraft attitude control for pointing as well as momentum desaturation.[9–13] Extensive research works are available about using magnetic torquers, and a few of them are introduced in Refs. 9–13.

For the inclined orbit operation, there are three essential requirements[1]: 1) ground antenna to track the satellite, 2) minimization of daily equivalent isotropically radiated power (EIRP) variations, and 3) compensation of rf polarization plane rotation. The compensation of rf polarization and the minimization of EIRP variations are the primary goals of attitude maneuvers.

The proposed method can be applied effectively with relatively light impact on spacecraft operation. Operation of the attitude maneuvers is assumed to be conducted by a ground station. The command itself is updated at regular time intervals to account for the ground operation scenario. This ground operation capability will constitute a complement of the existing onboard attitude control logics designed to provide continuous offset attitude maneuvers for the inclined orbit operation.

II. Attitude Perturbations in an Inclined Orbit

The sun/moon attraction over a satellite generally creates inclination angle change.[1] The nominal orbit inclination angle change rate for a satellite at the geostationary orbit is about 0.8 deg/year. If there is no north/south stationkeeping maneuver, the inclination angle would continue to grow with a period of about 55 years. Geometry of a satellite in an inclined orbit is provided in Fig. 1. From Fig. 1, the nonzero inclination angle results in longitude and latitude variations of the beam point on the Earth's surface that satisfy[7]

$$\sin\delta_{\text{lat}} = \sin i \sin\omega_0 t \tag{1}$$

$$\cos i = \tan(\omega_0 t + \delta_{\text{lon}})\cot\omega_0 t \tag{2}$$

These expressions can be approximated for small i and δ_{lon} and δ_{lat} as

$$\delta_{\text{lat}} = i\sin\omega_0 t \tag{3}$$

$$\delta_{\text{lon}} = -(i^2/4)\sin 2\omega_0 t \tag{4}$$

where ω_0 is the constant orbital rate (2π rad/day). The longitude and latitude variations produce a ground trace of figure-eight configuration.[7] The beam point variations alternatively can be related to the spacecraft attitude changes. For instance, the latitude change turns into roll attitude perturbation expressed as

$$\Delta\phi(t) = \sin^{-1}\left[\frac{r_e\sin\delta_{\text{lat}}}{\sqrt{r_e^2 + a^2 - 2ar_e\cos\delta_{\text{lat}}}}\right] \tag{5}$$

where r_e is the radius of the Earth and a is the semimajor axis of the orbit. On the other hand, the longitudinal variation causes the pitch angle change of the spacecraft, which is expressed as

$$\Delta\theta(t) = \sin^{-1}\left[\frac{r_e\sin\delta_{\text{lon}}}{\sqrt{r_e^2 + a^2 - 2ar_e\cos\delta_{\text{lon}}}}\right] \tag{6}$$

Note that the magnitude of the pitch angle change is relatively smaller than the roll change due to the smaller longitudinal variation. Finally, the yaw angle perturbation due to the nonzero inclination angle is approximated as[1]

$$\Delta\psi(t) = i\cos\omega_0 t \tag{7}$$

That is, the yaw angle is maximum being equivalent to the inclination angle at the nodal crossing point assuming that the time t is referenced to the instant when the spacecraft passes the node point.

To point a fixed target on Earth, the attitude changes caused by the inclined orbit in Eqs. (5–7) need to be compensated by attitude control. The attitude variations are not caused by external-disturbance-type torques, but they represent attitude pointing requirements for the spacecraft at the inclined orbit. In other words, the spacecraft should have offset attitude commands corresponding to $-\Delta\theta$, $-\Delta\phi$, and $-\Delta\psi$ about each body axis. Thus, the attitude variations with an opposite sign become reference trajectories for the spacecraft to track. The net effect will be fixing the beam point onto the target.

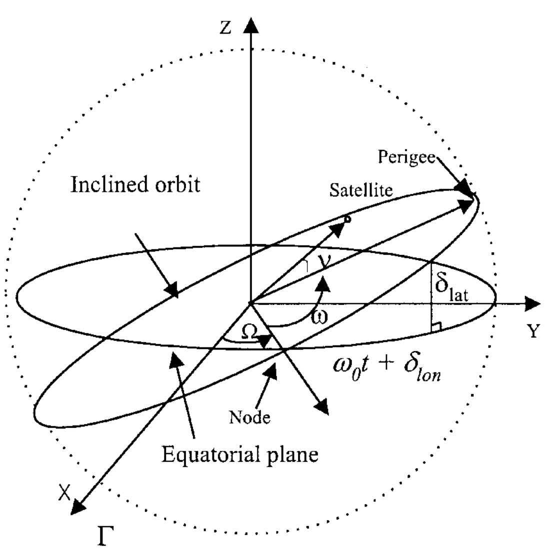

Fig. 1 Satellite orbiting in an inclined orbit.

Computation of Satellite Position

The current satellite location in the orbit must be identified before the reference attitude trajectories are generated. This is because the reference attitudes are functions of the elapsed time from the ascending node. Usually satellite orbit determination is performed by ground stations. Different forms of orbit elements are available by reformulation of the orbit determination results.[8] One popular choice is to take classical orbit elements given as a, e, i, Ω, ω, and ν as shown in Fig. 1, where a represents semimajor axis, e eccentricity, i orbit inclination angle, Ω right ascension of ascending node, ω argument of perigee, and ν true anomaly.[8]

By the assuming that the orbit elements are obtained at one instant and propagated continuously, the elapsed time t, which is measured from the ascending node crossing moment, can be computed from

$$t = t_c - t_n = (\omega + \nu)/\omega_0 \tag{8}$$

where t_c is the current time, t_n the time when the satellite crosses the ascending node, and ω_0 the orbital rate. The expression in Eq. (8) is based on the fundamental characteristics of the geostationary orbit, which is close to a circular shape.

Thus, for given orbital elements, especially the argument of perigee ω and the true anomaly ν, the satellite location and associated attitude data can be generated a priori. The initial orbit determination results are used to propagate ephemeris information, as well as time varying reference attitudes. For instance, if $a = 42166.9$ km, $e = 0.00007$, $i = 2.0$ deg, $\Omega = 235.28$ deg, $\omega = 23.05$ deg, and $\nu = 200.94$ deg, then the elapsed time is calculated as 0.622 days (53,757 s) from Eq. (8). The reference trajectories based on the initial condition are computed over two orbital periods and are plotted in Fig. 2. The classical two-body problem is numerically solved in the Earth centered inertial frame. The orbit propagation results are converted to the classical orbit elements to apply the results to Eq. (8).

Note that the reference pitch angle is multiplied by 50 for visualization purposes. Also, the pitch response has twice the frequency of the roll and yaw responses. This is also observed from Eqs. (3) and (4).

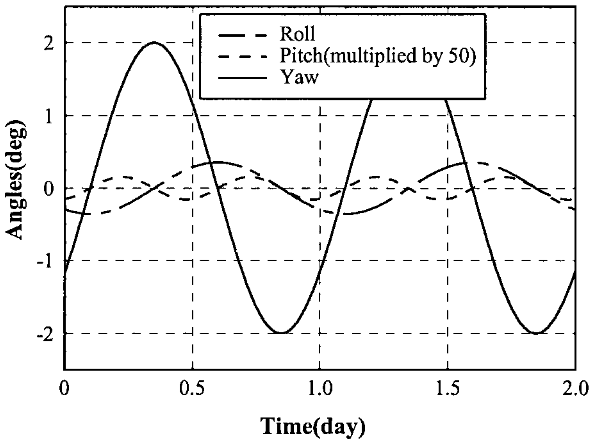

Fig. 2 Example reference attitude trajectories.

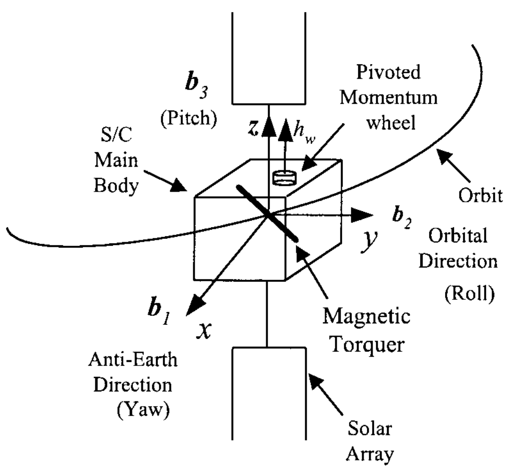

Fig. 3 Geometric configuration of a pitch bias momentum spacecraft.

III. Attitude and Actuator Dynamics

Before we discuss the control strategy, a brief overview on the attitude dynamics of pitch bias momentum spacecraft is presented. It also includes actuator dynamics of the pivot mechanism as well as the roll/yaw magnetic torquer.

Spacecraft Dynamics Equation

The simplified schematic configuration of pitch bias momentum spacecraft is presented in Fig. 3.

For the pitch bias momentum spacecraft, the bias momentum created by a spinning wheel exists along the orbit normal (or pitch) direction only. This creates gyroscopic coupling between the roll and yaw axes in the form of so-called nutational motion.[7,13] The pitch bias momentum is a typical bias momentum spacecraft with three-axis on-orbit attitude control capability in conjunction with a pivot steering mechanism, a magnetic torquer, and onboard thrusters.[1] The pivot mechanism possesses a rotational degree of freedom about the roll y axis with rotation angle γ. The pivot actuation is combined with the wheel angular momentum to generate yaw torque by gyroscopic principle.[13]

To derive governing equations of motion, we start with the total angular momentum vector of the system defined as

$$\boldsymbol{H} = (I_x\omega_x + h\sin\gamma)\boldsymbol{b}_1 + I_y\omega_y\boldsymbol{b}_2 + (I_z\omega_z + h\cos\gamma)\boldsymbol{b}_3 \tag{9}$$

where I_x, I_y, and I_z are body-axes principal moments of inertia, ω_x, ω_y, and ω_z are body-axes components of angular velocity vectors, and h is the angular momentum of the wheel. Application of Euler's equation into Eq. (9) yields

$$\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{H} + \hat{\omega}\times\boldsymbol{H} = \boldsymbol{T} \tag{10}$$

where $\hat{\omega} = \omega_x\boldsymbol{b}_1 + \omega_y\boldsymbol{b}_2 + \omega_z\boldsymbol{b}_3$ is the angular velocity vector and $\boldsymbol{T} = [T_x, T_y, T_z]^T$ is external input torque vector. The external torques consist of components from thrusters, the magnetic torquer, and space environmental disturbances as well. In this study, $\boldsymbol{T}$ is limited to the input by the magnetic torquer. The result of Eq. (10) can be described further as

$$\begin{aligned} I_x\dot{\omega}_x + (I_z - I_y)\omega_y\omega_z + \dot{h}\sin\gamma + h\dot{\gamma}\cos\gamma + h\omega_y\cos\gamma &= T_x \\ I_y\dot{\omega}_y + (I_x - I_z)\omega_z\omega_x + h(\omega_z\sin\gamma - \omega_x\cos\gamma) &= T_y \\ I_z\dot{\omega}_z + (I_y - I_x)\omega_x\omega_y + \dot{h}\cos\gamma - h\dot{\gamma}\sin\gamma - h\omega_y\sin\gamma &= T_z \end{aligned} \tag{11}$$

The inertia properties and angular momentum are taken to be $I_x = 16{,}533$, $I_y = 16{,}374$, and $I_z = 3{,}565$ in.-lbf$\cdot$ s^2, and $h = 475$ in.-lbf, which are those of KOREASAT, a typical midsize geostationary-Earthorbit communication satellite. The angular momentum is a nominal value and is usually adjusted continuously by the pitch control loop. On the other hand, the body axes components of the angular velocities are related to the 3–2–1 Euler angles as[7,13]

$$\begin{aligned} \omega_x &= -\dot{\theta}\sin\phi + \dot{\psi} - \omega_0\sin\phi \\ \omega_y &= \dot{\theta}\sin\psi\cos\phi + \dot{\phi}\cos\psi + \omega_0\sin\psi\cos\phi \\ \omega_z &= \dot{\theta}\cos\psi\cos\phi - \dot{\phi}\sin\psi + \omega_0\cos\psi\cos\phi \end{aligned} \tag{12}$$

where the Euler angles ϕ, θ, and ψ represent roll, pitch, and yaw angles, respectively, with respect to the orbiting local vertical local horizontal frame.[7] Substitution of Eqs. (12) into Eqs. (11) followed by small-angle approximation yields

$$\begin{aligned} I_x\ddot{\psi} + \left[(I_z - I_y)\omega_0^2 + \omega_0 h\right]\psi + h_g\dot{\phi} + \dot{h}\gamma + h\dot{\gamma} &= T_x \\ I_y\ddot{\phi} + \left[(I_z - I_x)\omega_0^2 + \omega_0 h\right]\phi - h_g\dot{\psi} &= T_y \\ I_z\ddot{\theta} + \dot{h} &= T_z \end{aligned} \tag{13}$$

where $h_g \equiv (-I_x - I_y + I_z)\omega_0 + h$ is introduced for notational simplicity. Note that the pivot angular motion $\dot{\gamma}$ multiplied by the wheel angular momentum generates yaw input torque $h\dot{\gamma}$. The linearized governing equations consist of coupled roll/yaw, x and y, dynamics and pitch z dynamic. The pitch dynamic is independent of roll/yaw dynamics, as is fairly well known. The roll/yaw dynamics are usually described as a combination of nutational and orbital motion. The natural frequencies of the roll/yaw dynamics from Eq. (13) are given by[13]

$$\omega_0, \qquad \sigma \equiv \sqrt{h_g^2 / I_x I_y} \tag{14}$$

where σ is the nutational mode frequency[13] due to the nonzero angular momentum of the wheel. The nutational mode period for KOREASAT is about 220 s for the nominal angular momentum. The nutational motion is usually dominant for the short period of time and controlled by the pivot and/or the magnetic torquer control logic.

On the other hand, control torque by a magnetic torquer is generated by the interaction of the Earth's magnetic field and the magnetic dipole moment of the torquer.[13,14] Mathematically, it is stated as

$$\boldsymbol{T} = \boldsymbol{M} \times \boldsymbol{B} \tag{15}$$

where $\boldsymbol{B} = B_x\boldsymbol{b}_1 + B_y\boldsymbol{b}_2 + B_z\boldsymbol{b}_3$ is the Earth's magnetic field vector and $\boldsymbol{M}$ is the magnetic dipole moment vector about the spacecraft body axes. The components of the magnetic field vector vary depending on the orbital longitude and latitude.[13,14] Usually at geostationary orbit, the pitch component B_z is known to be dominant over other components.[14] Thus, the magnetic field vector can be approximated as

$$\boldsymbol{B} \simeq B_z \boldsymbol{b}_3 \tag{16}$$

The preceding approximation is useful not only in the sense of the relative magnitude, but also the original objective of using the magnetic torquer. In this study, the magnitude of the magnetic field vector is estimated as $B_z = 104.52$ nT (9.25×10^{-7} in.-lb/atm^2). In addition, the magnetic dipole moment can be expressed about the spacecraft body axes. The magnetic torquer of KOREASAT, as is GSTAR III, is skewed about the roll/yaw body axes. Thus, the dipole moment vector about the roll/yaw axes is expressed as

$$\boldsymbol{M} = -M \sin\beta \boldsymbol{b}_1 + M \cos\beta \boldsymbol{b}_2 \tag{17}$$

where β is the skew angle of the torquer. The magnitude of the dipole moment is 300 atm^2 while the skew angle is 60 deg to achieve balanced performance in roll/yaw control. The final expression of the magnetic torque, therefore, becomes

$$\begin{aligned} \boldsymbol{T} &= M \cos\beta B_z \boldsymbol{b}_1 + M \sin\beta B_z \boldsymbol{b}_2 \\ &= T_x \boldsymbol{b}_1 + T_y \boldsymbol{b}_2 \end{aligned} \tag{18}$$

There is no magnetic torque component acting over the pitch axis because it is actually so small. The small pitch magnetic torque is in fact controlled by the momentum wheel speed modulation. Therefore, the magnetic torquer is exclusively used for roll/yaw control either by ground-loop or onboard control

IV. Attitude Maneuver Strategies

Based on the attitude perturbation analysis and attitude dynamics with actuators included, attitude maneuver strategies for pointing in an inclined orbit are developed. Independent controls over attitude perturbations about three axes are proposed in conjunction with simulation analysis. The attitude control itself is assumed to be operated by ground-loop control, but can be extended into onboard implementation. The ground-loop control is more conservative than the onboard approach especially in the sense of ground processing time delay and potential noise in the transmission of command and telemetry signals.

Pitch Control

For pitch pointing, the wheel speed is continuously adjusted about the nominal angular momentum depending on the pitch error signal sensed by the Earth sensor. The nominal pitch control loop is normally closed, and the spacecraft is locked onto the Earth about the pitch axis. The pitch loop is controlled by an onboard processor. The onboard processor can be programmed for time-varying pitch offset angles within a certain range. The wheel speed can also be combined with a constant bias command from the ground station. The bias wheel speed command is uplinked by ground commanding systems.

Because the goal of pitch control is to adjust the wheel speed so that the actual pitch angle counteracts the reference pitch angle being equivalent to the perturbation effect, we propose here to generate a wheel speed command at a regular time interval. The original onboard closed-loop pitch control command is

$$\dot{h} = K_P(\theta + K_D \dot{\theta}) \tag{19}$$

where K_P and K_D are typical design parameters to be selected. Stability of pitch axis is maintained by the control law of Eq. (19). The reference pitch command constructed on the ground is prescribed to be

$$\dot{h}_{\text{ref}} = -K_P(\theta_{\text{ref}} + K_D \dot{\theta}_{\text{ref}}) \tag{20}$$

where θ_{ref} is associated with the reference pitch angle due to nonzero inclination angle. The reference wheel torque command is added as

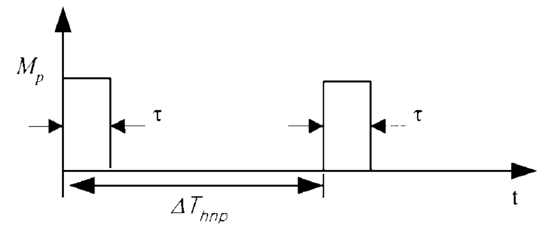

Fig. 4 Pivot commands over half-nutation period.

a bias command to the original feedback wheel torque command of Eq. (19). Therefore, the total command to be applied to the wheel becomes

$$\dot{h} + \dot{h}_{\text{ref}} = K_P[\theta - \theta_{\text{ref}} + K_D(\dot{\theta} - \dot{\theta}_{\text{ref}})] \tag{21}$$

Note that the reference pitch command, therefore, translates into an error feedback between the actual and reference pitch angles. The reference pitch trajectory θ_{ref} is selected as $-\Delta\theta$ in Eq. (6) to counteract the pitch axis deviation due to the orbit inclination angle. The reference command is issued at a regular time interval. After trials of different values, the pitch reference command update interval is selected as 20 s, which takes into account ground processing time with the 2-s telemetry transmission rate.

Roll Pointing by Pivot Mechanism

The momentum wheel with the pivot mechanism is a key attitude control subsystem for the pitch bias momentum satellite. The pivot angular rate command produces yaw torque, as seen from Eq. (13). The pivot torque is applied to controlling the nutational motion caused by the angular momentum of the wheel, and it can also produce a constant roll offset command for the pointing maneuver. The desired pivot steps in this case can be commanded from the ground station.

Let us consider pivot command pulses applied at different time instants that are separated by a half-nutation period as in Fig. 4, where ΔT_{hnp} is the half-nutation period. The ideal pivot command needs to be an impulsive type, but from a practical consideration, the command has a finite duration (Fig. 4). With the pivot actuation duration τ, the first input command in Laplace domain can be expressed as

$$\gamma(s) = (M_p/s)(1 - e^{-\tau s}) \tag{22}$$

where $M_p = -h\dot{\gamma}$ is the magnitude of the yaw torque produced by the pivot steering. Equation (22) can be approximated for a reasonably small τ as

$$\gamma(s) \simeq M_p \tau \tag{23}$$

To examine the pivot command applied, Eq. (13) is further approximated for the pure rigid and nutational motion in roll/yaw dynamics. The smaller coefficients of angular variables ϕ and ψ containing orbital rate ω_0 are dropped because we are interested in short-term nutational dynamics. The approximated dynamics become

$$I_x \ddot{\psi} + h_g \dot{\phi} + h\dot{\gamma} = T_x, \qquad I_y \ddot{\phi} - h_g \dot{\psi} = T_y \tag{24}$$

where we also neglected $\dot{h}\gamma$ for small γ from Eq. (13). After application of the first pivot command into Eq. (24), it is not difficult to show

$$\phi(t) = \phi_c + (M_p \tau / h_g)(1 - \cos\sigma t) \tag{25}$$

$$\psi(t) = \psi_c + (1/I_x)(M_p \tau/\sigma) \sin\sigma t \tag{26}$$

where ϕ_c and ψ_c represent the constant roll and yaw angles, respectively, before the application of the first pivot command. It is easy to observe that the roll and yaw responses translate into the nutational motion with 90-deg phase difference. Now the second pivot impulse input in Fig. 4 is augmented so that Eqs. (25) and (26) become

$$\begin{aligned} \phi(t) &= \phi_c + (M_p\tau/h_g)(1 - \cos\sigma t) + (M_p\tau/h_g)[1 - \cos(\sigma t - \pi)] \\ &= \phi_c + 2M_p\tau/h_g \end{aligned} \tag{27}$$

and

$$\psi(t) = \psi_c + (1/I_x)(M_p\tau/\sigma)\sin\sigma t + (1/I_x)(M_p\tau/\sigma)\sin(\sigma t - \pi)$$
$$= \psi_c \tag{28}$$

As we can see, the yaw angle remains unchanged at the result of the two consecutive pivot commands, while the roll angle comes to an offset displacement. By using Eq. (27) and the relationship $M_p = -h\dot{\gamma}$ with the approximation $h_g \simeq h$, we can derive

$$\dot{\gamma} = (\phi_c - \phi)/2\tau \tag{29}$$

Therefore, Eq. (29) shows a relationship between the resultant roll angle and the associated pivot rate command. On the other hand, for the roll pointing objective in an inclined orbit, the roll command should be generated in such a way that the actual roll angle follows the reference roll angle. Thus, the reference roll angle is incorporated into Eq. (29) so that

$$\dot{\gamma} = (\phi_c - \phi_{\text{ref}})/2\tau \tag{30}$$

where the reference roll angle ϕ_{ref} is taken as $-\Delta\phi$, introduced in Eq. (5), in the same manner as the pitch control case. Note that if the pivot angular rate $\dot{\gamma}$ is fixed then the command duration τ could be adjusted so that Eq. (30) is automatically satisfied. The required pivot command angle is, consequently, expressed as

$$\gamma(t) = \gamma(0) + 2\int_0^\tau \dot{\gamma}\,\mathrm{d}t \tag{31}$$

Note that the roll pointing maneuver does not have to be conducted continuously due to practical considerations such as ground processing time, telemetry transmission rate, and finite pivot actuation delay. Instead, it could be performed at a certain time interval as long as the pointing requirement is still satisfied between the pivot actuation instants. The pivot actuation command is applied at every half-hour in this study. It can be expected that the tracking performance improves with the shorter control update interval. The half-hour update interval is also a conservative choice considering the minimum ground processing time required, which is usually much shorter.

Simulation based on the pitch control logic in Eq. (21) and the roll control logic in Eq. (29) has been conducted with the original nonlinear governing equations of motion. The resultant angular responses together with the reference trajectories are presented in Fig. 5. Some initial errors between the reference and actual states are introduced to reflect the uncertain initial states. As shown, the pitch angle tracks the reference trajectory fairly well. This is by virtue of the feedback strategy of the pitch control logic and the relatively short update interval. Meanwhile, the roll angle tracks the reference trajectory within a certain pointing accuracy by the pivot control logic explained earlier. The roll angle remains almost constant over the interval where the pivot is not actuated. The pivot angular rate sign alternates over the whole maneuver period, as it can be seen in Fig. 6.

On the other hand, the yaw response in Fig. 5 shows a poor tracking performance because there is no direct control over the yaw angle. The yaw angle is maximum at the node, being a principal parameter causing the rf polarization plane rotation.[1] A separate control action, therefore, needs to be taken to control the yaw angle. The independent control over the yaw angle using the magnetic torquer is discussed next.

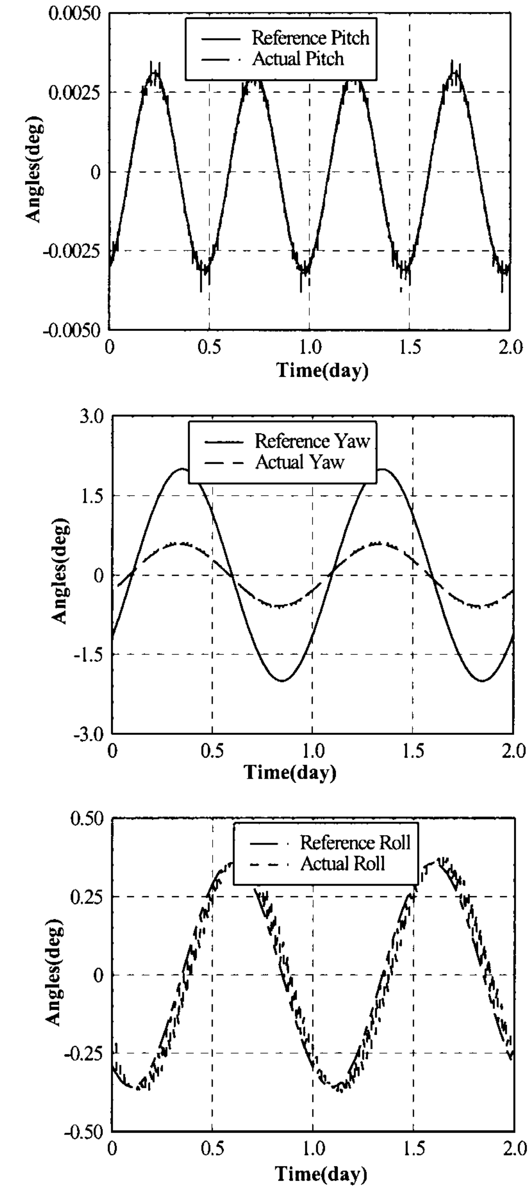

Fig. 5 Simulation results by applying the pitch and roll actuation logics.

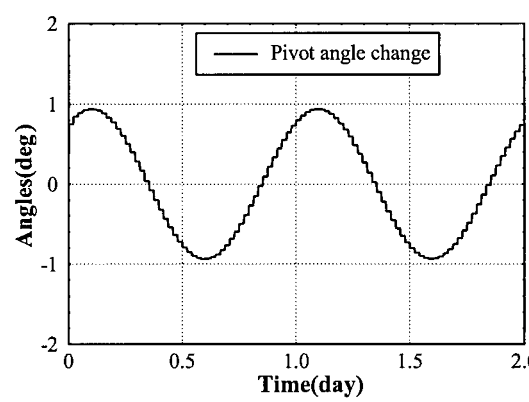

Fig. 6 Pivot angular displacement change during the roll pointing.

Yaw Pointing by Magnetic Torquer

There is no direct control over the yaw for the general pitch bias momentum spacecraft. Instead, roll control automatically turns into the yaw control due to the inherent roll/yaw coupling over the orbital period. The roll data measured by the Earth sensor is primarily supplied to the roll control logic. Such an indirect approach of yaw control by the roll/yaw coupling effect, however, is not sufficient for the inclined orbit operation. Additional yaw control mechanisms need to be designed. One possibility is to use the magnetic torquer, which is also a primary control device controlling roll/yaw errors during the normal operation mode of GSTAR III and KOREASAT. The primary function of the torquer is to produce a constant roll offset angle as well as to control small post stationkeeping nutational motion. The input to the torquer control logic is roll angle data from the Earth sensor.

The magnetic torquer can be used to control either yaw or roll angle on the availability of information. From Eq. (18), we can observe almost an equivalent level of control authority about yaw and roll axes by the magnetic torquer. To control the yaw angle directly, however, the yaw angle data should be provided to the control logic. If the yaw angle becomes available, the magnetic torquer can directly control the yaw angle in a manner similar to the roll control counterpart. In Ref. 1, Parvez and Misra did not consider using the magnetic torquer and instead used onboard thrusters to reduce the maximum yaw angle at the node.

The yaw control logic, however, assumes that the yaw data are available in real time, which is not generally true because the Earth sensor measures pitch and roll angles only. The yaw angle data are measured during stationkeeping maneuvers only by a rate measurement unit for KOREASAT. The yaw angle, therefore, needs to be estimated by an indirect approach. A real-time ground estimator is proposed as is explained later.

Based on the yaw angle availability, the present magnetic torquer logic, which is targeted to direct roll control, is slightly modified. The present roll control logic commands an opposite dipole moment in the electromagnet depending on the polarity of the roll error. The phasing of the torquer switching is controlled by setting the time delay t_d, which is equivalent to the half-nutation period. The dipole is activated t_d seconds after the magnitude of the averaged roll signal exceeds the roll threshold. To provide some damping, the dipole remains on until t_d seconds after the roll signal changes sign. The magnetic torquer switching changes the polarity M of the torquer, which in turn results in the roll/yaw torque inputs. The magnetic torquer command update interval is set to be 16 s.

The new control variable defined as an error between the reference and estimated yaw angles replaces the roll angle for the yaw tracking purpose. That is,

$$\psi_{\text{error}} = \psi_{\text{ref}} - \hat{\psi} \tag{32}$$

where ψ_{error} is the yaw error determining the torquer polarity, ψ_{ref} the reference yaw angle, and $\hat{\psi}$ the estimated yaw. The reference yaw is taken as $-\Delta\psi$, opposite to the yaw angle perturbation of Eq. (7). Because the estimated yaw angle is needed for the implementation of the yaw control logic, detailed discussion associated with the yaw angle estimation is presented in the following section.

The overall block diagram of the proposed ground-loop control system including the estimator discussed is presented in Fig. 7.

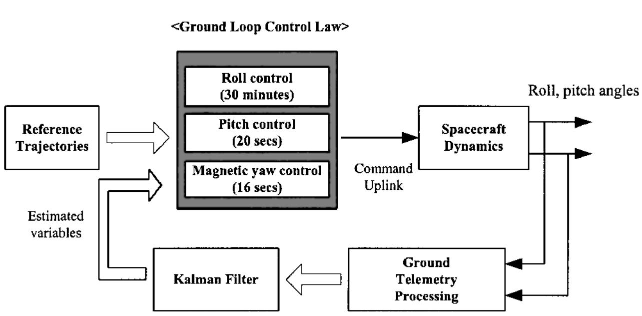

Fig. 7 Block diagram representation of the overall ground-loop control system.

Estimation of Attitude Variables

As already mentioned, the magnetic torquer needs yaw angle information for direct yaw pointing. The Earth sensor, however, does not measure the yaw angle directly. The yaw angle information may be obtained by a state estimation technique taking the measured roll angle as input data. Other variables such as roll and pitch angles are also used to construct required control commands. Also we have already assumed that the pitch command by the wheel and the roll command using the pivot are constructed on the ground. This process requires knowledge of roll and pitch angles with reasonable accuracy. The roll and pitch angles are measured by Earth sensor and are transmitted regularly to the ground station. The transmitted signals are usually subject to various noise sources. Therefore, a practical issue in the implementation of the proposed algorithms is to guarantee smoothed data on a continuous basis. This requirement naturally leads us to the yaw angle estimation. Also, the wheel angular momentum is another parameter that can be estimated by using the data from the telemetry information.

In this study, the yaw angle is estimated continuously based on the real-time telemetry data at the transmission rate of every 2 s. The roll/pitch angles and wheel angular momentum are also estimated in the sense of smoothing the noisy telemetry data. A single filter is designed for this objective. The estimation technique adopted is the typical nonlinear extended Kalman filter (EKF) algorithm.[15]

Design of the EKF starts from Eqs. (11) and (12), which can be combined in state-space form as

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(t, \boldsymbol{x}) + \boldsymbol{\xi} \tag{33}$$

where $\boldsymbol{\xi}$ is a vector of plant disturbance modeled as white noise and $\boldsymbol{x}$ a state vector defined as

$$\boldsymbol{x} = [\psi, \phi, \theta, \omega_x, \omega_y, \omega_z, h]^T \tag{34}$$

The angular momentum of the wheel is included in the state vector because the telemetry information is available for h, which is usually corrupted by noise.

The output measurement variables consist of pitch/roll angles from the Earth sensor and the angular momentum of the wheel:

$$\boldsymbol{y} = \boldsymbol{h}(\boldsymbol{x}) \equiv \begin{Bmatrix} \theta \\ \phi \\ h \end{Bmatrix} + \boldsymbol{\eta} \tag{35}$$

where $\boldsymbol{\eta}$ is a vector of white measurement noise. The measurement interval, which corresponds to the telemetry transmission rate of KOREASAT, is 2 s. The filter update interval is also selected as 2 s, sufficient for ground processing of the filter algorithm in real time.

Next the EKF algorithm is applied to Eqs. (33) and (35) by assuming white noise in the system dynamics and measurement equation. The sensor noise data are obtained from a sensor design book. The state estimate update at a given step is prescribed by[15]

$$\hat{\boldsymbol{x}}_e(+) = \hat{\boldsymbol{x}}_e(-) + L\{\boldsymbol{y} - \boldsymbol{h}[\hat{\boldsymbol{x}}_e(-)]\} \tag{36}$$

where L is the Kalman filter gain obtained by the EKF algorithm, $(-)$ represents a state before update, and $(+)$ represents a state after update. The plant disturbance and measurement noise covariance matrices are given as

$$E[\boldsymbol{\xi}\boldsymbol{\xi}^T] = \begin{bmatrix} 10^{-16} I_{3\times 3} & 0 & 0 \\ 0 & 10^{-18} I_{3\times 3} & 0 \\ 0 & 0 & 10^{-3} \end{bmatrix}$$

$$E[\boldsymbol{\eta}\boldsymbol{\eta}^T] = \begin{bmatrix} 7.925 \times 10^{-8} & 0 & 0 \\ 0 & 7.925 \times 10^{-8} & 0 \\ 0 & 0 & 0.1 \end{bmatrix}$$

where $I_{3\times 3}$ is a 3×3 unit matrix and E is the expectation of random variables.[15] The relatively large number in disturbance covariance for the angular momentum is because of the low resolution in converting the analog data into a digital format for transmission from spacecraft to the ground. The disturbance noise covariance matrix is selected by trial and error, which yields satisfactory filter performance in conjunction with the sensor noise covariance.

The roll/yaw dynamic coupling due to the nutational and orbital motion is a key factor determining the performance of the estimator especially for the yaw estimation. The stronger coupling effect usually speeds up the filter convergence. The nonlinear Kalman filter algorithm has been implemented and tested through simulation. The initial state error covariance matrix was selected after a few trials. For simulations of the designed filter performance, random measurement noise is included to simulate a realistic ground control scenario. The initial attitude errors are arbitrarily assumed as $\phi(0) = 0.5$ deg, $\psi(0) = -0.3$ deg, and $\theta(0) = 0.05$ deg, respectively, while the initial conditions for the estimator are set to zero. Results of the filter run for the first 2 h are presented in Fig. 8. The angular errors between the estimator and actual dynamics are plotted.

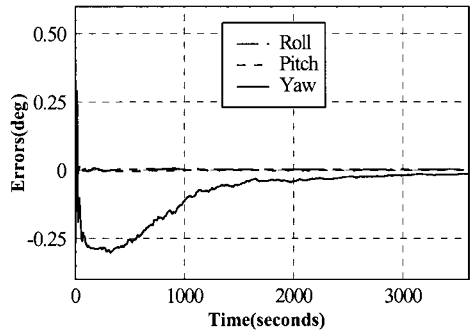

Fig. 8 Simulation results of the Kalman filter run.

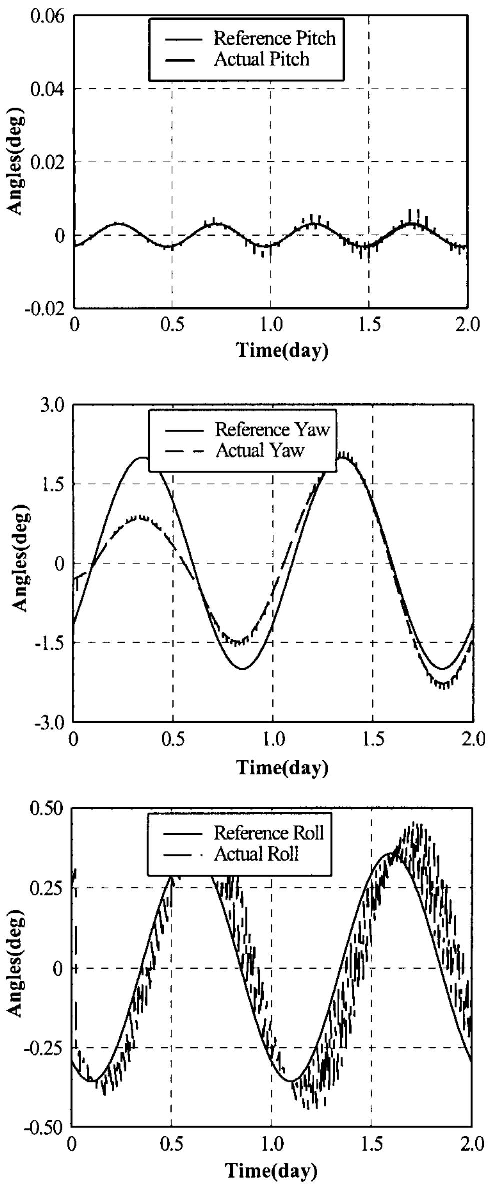

Fig. 9 Pointing simulation using the estimated attitude variables.

The Kalman filter shows quick convergence in roll and pitch, as it should, inasmuch as the roll and pitch angles are directly measured by an Earth sensor with noise. The yaw angle convergence is slow, however, because it is not directly measured but is estimated in conjunction with the roll measurement.

Pointing with Estimated Variables

A ground-based Kalman filter design was discussed in the preceding section. The filter estimates attitude angles and the wheel angular momentum. The estimated yaw angle was used to determine the polarity of the magnetic torquer [Eq. (32)]. For ground-loop implementation of the roll and pitch commands, the estimated roll and pitch angles could be employed. In particular, the estimated roll angle is applied to constructing the pivot actuation command of Eq. (30). Therefore, Eq. (30) can be rewritten as

$$\dot{\gamma} = (\hat{\phi}_c - \phi_{\text{ref}})/2\tau \tag{37}$$

where $\hat{\phi}_c$ is now the estimated current roll angle. With this modification, the pointing maneuver is reconstructed by utilizing the estimated variables. The original pointing algorithms remain the same with only the estimated attitude variables introduced. Initial errors between the estimator and actual dynamics are employed for simulation purpose. Simulation results are presented in Fig. 9.

As seen from Fig. 9, the simulation results demonstrate the effectiveness of the ground-based estimator for the desired pointing maneuver. The roll pointing performance is essentially similar to that of Fig. 5, which also illustrates satisfactory filter performance. Despite relatively slow convergence in the yaw response, the yaw tracking maneuver exhibits significant improvement over the result of Fig. 5. In spite of the small torque level, the magnetic torquer is effective in tracking the reference yaw with a maximum value of 2 deg.

The maneuver performance is generally subject to the magnitude of the inclination angle, however, which is also related to the physical limitation of the maximum pivot angle allowed. Also, the limitation of the Earth sensor field of view in the geostationary mission orbit is another factor to be taken into account to extend the results of this study. The ground-loop approach obviously implies other alternatives by using onboard processors, including flight software reconfiguration for the case of a spacecraft with an onboard computer available.

V. Conclusions

An attitude control technique for a geostationary pitch bias momentum satellite in an inclined orbit is demonstrated with simulation verifications. The geometry of the inclined orbit is used to formulate spacecraft attitude perturbation effects. The attitude perturbations were compensated for by a momentum wheel with a pivot device and roll/yaw magnetic torquer actuators. The control commands were assumed to be applied by a ground-loop control system at regular time intervals. The yaw angle data needed for the magnetic torquer control were provided by a ground-based estimator. The relatively slow convergence rate in yaw response turned out to have a rather insignificant impact on the simultaneous roll/yaw pointing. The accuracy of yaw angle estimation may be improved when other additional sensors are available, such as a sun sensor and star sensor measuring yaw angle directly.

The magnitude of inclination angle of 2 deg used could be further extended with increased maximum pivot displacement of the momentum wheel. The control strategies developed have potential applications to generic bias momentum spacecraft with similar configurations of actuators and sensors.

Acknowledgments

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